Calculating Average Path Length

Average path length is a concept in network topology that is defined as the average number of steps along the shortest paths for all possible pairs of network nodes. It is a measure of the efficiency of information or mass transport on a network. – Wikipedia

We can calculate average path length of a graph by using following formula:

\[ l_G = \frac{1}{n \cdot (n - 1)} \cdot \sum_{i \neq j} d(v_i, v_j) \]

Here \( d(v_i, v_j) \) represents the length of shortest path exists between two vertices. So, we take sum of all shortest paths between all vertices and divide number of all possible paths.

Example:

Consider graph shown in Figure 1. Now we calculate shortest path for each pair of vertices

\[
\begin{align*}
  d(1,2) &= 1 \\
  d(1,3) &= 1 \\
  d(1,4) &= 2 \\
  d(2,1) &= 1 \\
  d(2,3) &= 1 \\
  d(2,4) &= 2 \\
  d(3,1) &= 1 \\
  d(3,2) &= 1 \\
  d(3,4) &= 1 \\
  d(4,1) &= 2 \\
  d(4,2) &= 2 \\
  d(4,3) &= 1
\end{align*}
\]

By putting in formula, we get:

\[
\begin{align*}
  l_G &= \frac{1}{4(4 - 1)} \cdot (1 + 1 + 2 + 1 + 1 + 2 + 1 + 1 + 2 + 2 + 1) \\
  &= \frac{1}{12} \cdot 16 \\
  &= \frac{4}{3} \\
  &= 1.33
\end{align*}
\]

You can verify by executing following lines in R:

```r
# Create a graph
g = graph(edges=c(1,2,1,3,2,3,3,4), directed=F)
# Calculate average path length
average.path.length(g)
```